paper VII Bsc part III
Group - A

Date 16/02/24
Page 1

"Saha Theory of Ionization" & Its two important Applications"

Dr. Usha Kumari

In addition to electronic impact & photoionisation, there are other processes which can cause excitation and ionization of atom. The important one of these process is the heating of the gas. At the lowest temperature, all the substances are in the solid state. When heat is added the substance passes to liquid state. When more heat is added, the liquid passes to gas phase. When the gas is further heated the molecules of the gas dissociate (i.e break into component parts) When still further heat is added the atoms collide so violently against each other that the valence electrons are knocked off and atoms get ionised. Just as in the case of electronic impact ionisation is proceeds by excitation and in the case of atoms, line spectra and band spectra in the case of molecules appear.

An approximate idia of temp. at which excitation can take place be calculated from the relation.

$$h\nu = KT$$

If the line emitted has the wavelength $\lambda = 5893\,Å$ then

$$\frac{6.57 \times 10^{-27} \times 3 \times 10^{10}}{5893 \times 10^{-8}} = 1.37 \times 10^{-16} T$$

$$\therefore T \approx 25\,000\,K$$

When the temp is still further increased. the outer electrons are detacted from the atom and ter ionision of Na.

$$eV_i = KT$$

or $$1.62 \times 10^{-12} \times 5.12 = 1.37 \times 10^{-16} T$$

$$\therefore T = 65\,000\,K.$$

In 1921 Dr. M. N Sahay advanced the theory of thermal ionisation and showed that the spectra of stars can best explained by assuming that in stars atoms are excited to higher states and ionised to successive degree by thermal ionisation.

To put the assumtion mathematically Dr. Saha considered Ca atom & neglecting the process of excitation & assuming only ionisation.

$$Ca \rightleftharpoons Ca^{+} + e \quad \text{—— (1)}$$

$$\therefore \partial \phi = 0 \quad \text{—— (2)}$$

where ϕ is thermodynamic potential.

$$\& \quad \phi = U + PV - TS \quad \text{—— (3)}$$

where U = internal energy
P = pressure
S = entropy
T = Temp.

$$\therefore \phi_{\alpha}^{+} \phi_{e} + \phi_{\alpha} = 0$$

where ϕ_{α}^{+}, ϕ_{e} & ϕ_{α} are thermodinamic potentials for ions electron and atom respectively

$$\therefore \quad \psi = -\frac{\phi}{T} - S - \frac{U + PV}{T}$$

$$\therefore \quad \psi_\alpha^+ + \psi_e - \psi_\alpha = 0$$

$$\text{or} \quad \psi_\alpha = \psi_\alpha^+ + \psi_e \quad \text{--- } (4)$$

$$\psi_a = C_{p_s} \log T - R \log P_a + R \log \left\{ \frac{(2\pi m)^{3/2} g_a k^{5/2}}{h^3} \right\} - \frac{U_a}{T}$$

where C_{p_a} = molecular specific heat

g_a = statistial weight factor for neutral atom.

U_a = energy at absolute zero.

m = mass of the atom.

$$C_p = \frac{5}{2} R$$

$$\therefore \quad \psi_a = \frac{5}{2} R \log T - R \log P_a + R \log \left[\frac{(2\pi m)^{3/2} k^{5/2} g_a}{h^3} \right] - \frac{U_a}{T}$$

$$\text{or,} \quad \frac{\psi_a}{R} = \frac{5}{2} \log T - \log P_a + \log \left\{ \frac{(2\pi m)^{3/2} k^{5/2} g_a}{h^3} \right\} - \frac{U_a}{RT}$$

Similarly for ψ_a^+

$$\frac{\psi_a^+}{R} = \frac{5}{2}\log T - \log P_a^+ + \log\left\{\frac{(2\pi m)^{3/2} K^{5/2} g_a^+}{h^3}\right\} - \frac{U_a^+}{RT} \quad \&$$

for the electron

$$\frac{\psi_e}{R} = \frac{5}{2}\log T - \log P_e + \log\left\{\frac{(2\pi m)^{3/2} K^{5/2} g_a^+}{h^3}\right\} - \frac{U_e}{RT}$$

Using eqn (4) we have

$$0 = -\frac{5}{2}\log T - \log P_a + \log P_a^+ P_e - \log\frac{(2\pi m)^{3/2} K^{5/2}}{h^3} + \log g_a - \log g_a^+ g_e - \frac{U_a^+ - U_a + U_e}{RT}$$

$$= -\frac{5}{2}\log T + \log\frac{P_a^+ P_e}{P_a} - \log\frac{g_a^+ g_e}{g_a} - \log\frac{(2\pi m)^{3/2} K^{5/2}}{h^3} + \frac{U_a - (U_a^+ - U_e)}{RT}$$

$$\therefore \log \frac{P_a^+ P_e}{P_a} = \frac{5}{2}\log T + \log \frac{(2\pi m)^{3} k^{5/2}}{h^3} + \log \frac{g_a^+ g_e}{g_a} - \frac{U}{RT} \quad (5)$$

where $U = U_a^+ + U_e - U_a$

$\therefore P_a^+ = nxkT = P_e$ & $P_a = nkT(1-x)$

& $P = P_a + P_a^+ + P_e$

or, $P = nkT(1-x) + nxkT + nxkT$

$= nkT(1+x)$

$\therefore P_a^+ = P_e = \left(\frac{x}{1+x}\right) P$ &

$P_a = \frac{(1-x)}{(1+x)} P$

$\therefore$ from equ^n (5) we have

$$\log\left(\frac{x^2}{1-x^2}\right)P = \frac{5}{2}\log T - \log\left\{\frac{(2\pi m)^{3/2} k^{5/2}}{h^3}\right\} + \log \frac{g_e g_a^+}{g_a} - \frac{U}{RT}$$

$$= \log\left\{\frac{T^{5/2}(2\pi m)^{3/2}K^{5/2}}{h^3}\right\} + \log\frac{g_e g_a^+}{g_a} - \frac{U}{RT}$$

$$\therefore \log\left[\frac{\left(\frac{x^2}{1-x^2}\right)P}{T^{5/2}(2\pi m)^{3/2}K^{5/2}g_e g_a^+}\right] = -\frac{U}{RT}$$

$$\therefore \left(\frac{x^2}{1-x^2}\right)P \Big/ \left(\frac{T^{5/2}(2\pi m)^{3/2}K^{5/2}g_e g_a^+}{g_a h^3}\right) = e^{-U/RT}$$

$$\therefore \left(\frac{x^2}{1-x^2}\right)P = \left(\frac{2\pi m}{h^2}\right)^{3/2}(KT)^{5/2}\frac{g_e g_a^+}{g_a}e^{-U/RT} \quad (6)$$

This is Saha's equation or ionisation formula.

— x —